Ομοκλινικές συνδέσεις και η επίδρασή τους στην ασφάλεια πλοίων

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Philosophy of approaches for predicting ship behavior at sea

Target: **Ordinary behavior**
Emphasis: **Accurate modeling**

*Propulsion/environmental efficiency economy, comfort*

Features:
- Quantitative – ship geometry
- “First principles” hydrodynamics
- Mildly nonlinear
- Few simulations
- Time domain analysis

Target: **Extremes, limits of safe behavior**
Emphasis: **Ship motion phenomena**

*Capsize, surf-riding, directional instability, parametric rolling, sudden heel during loading or due to symmetric water entry (loll), oscillations of moored vessels, sub-harmonic & chaotic motions*

Features:
- Intuitive modeling - low-dimensional model approximations
- Ideal for understanding effect of nonlinearities
- Holistic in the dynamics
- Good for safety criteria development
- Analysis in *phase space*

Combines:
- Ship hydrodynamics
- Nonlinear dynamics – stability theory
- Stochastic dynamics
The phase-space

Motions seen as a “flow” in the *phase-space*

For a nonlinear system, phase flow’s geometry can be *qualitatively changed* as some influential parameter is varied

- **Homoclinic**: phenomena triggered by the contact of *invariant manifolds* – formation of homoclinic or heteroclinic curves on Poincaré section of phase flow
- They modify globally the phase-space layout (change not in small neighborhood)
- Relate with *dynamic instabilities* – IMO’s 2nd Generation Ship Stability Criteria address homoclinic phenomena
- Can explain unusual ship motions

  Typical manifestations
  - abrupt disappearance of periodic behavior
  - connected with ship capsize
  - introduce motion irregularity

bifurcations
(Various types)

Homoclinic
Essential concepts introduced through a ship’s roll motion

Energy dissipating system

The basin (domain) of attraction

Externally excited system
Formation of *homoclinic* curves

**Homoclinic**

**Heteroclinic**

*before*  

**Autonomous system**  

*after*

periodic motion destroyed

**Homoclinic saddle connection**

**Externally excited system**

tangency

basin erosion

started

**Essential concepts (cont’d)**
Prediction tools

**Direct simulation** (= blind search)

**Continuation** (path - following): suitable for deterministic systems  

**Melnikov’s method**: semi-analytical - monitoring distance of invariant manifolds (for deterministic and stochastic systems)  
Works also as “energy balance” (Thompson 1997)*

**Lagrangian coherent structures**: field methods

- **Finite-time Lyapounov exponents (FTLE)**  
  *Haller & Yuan (2000), Shadden, (2011) …*

- **Finite-size Lyapounov exponents (FSLE)**  
  *Artale et al. (1997), Koh & Legras (2002) …*

- **Variational method**  
  *Haller (2011)*

- **Extraction of coherent sets via clustering**  
  *Froyland & Padberg-Gehle 2015*
Summary of *homoclinic* phenomena in ship motions

**Unforced ship:**
→ Heteroclinic saddle connection modifies the **boundary of free rolling**, as heel bias is strengthened.

**Regular waves:**
- Beam-seas: Homoclinic and/or heteroclinic intersections enable **capsize**
- Following/quartering seas:
  → Homoclinic saddle connection **leads to global surf-riding and possibly to broaching-to**
  → Homoclinic saddle connection **ends oscillatory surf-riding** on wave down-slope.

**Multi-chromatic/stochastic waves:**
- Following/quartering seas:
  → Heteroclinic intersections increase susceptibility for **broaching-to**.

**Wind loading:**
- Beam wind
  → Homoclinic to saddle-node (omega explosion): barrier between **straight-line motion** and **turning**.
- Head wind:
  → Homoclinic saddle connection **ends yaw oscillations** (ship heading into wind with sluggish rudder control.)
The roll motion

a) Unforced case

Basic (normalized) model for rolling with heel bias

\[ \ddot{x} + b_1 \dot{x} + b_3 x^3 + x - (1-a) x^2 - ax^3 = f \sin \Omega t \]

Melnikov prediction: heteroclinic curve formed at:

\[ a = 1 - \sqrt{2} b_1 - \frac{12 \sqrt{2}}{35} b_3 \] (error <1%)

The "weather criterion"

- wind
- wave
- steady heel
- damped resonant rolling reduced for irregularity of waves

off-equilibrium windward angle of release \( \theta_0 \) (in still-water)

Restoring and heeling lever

(2002) JSR, 46/3
b) With harmonic forcing

Safe basin split into regions of small and large motion

Homoclinic intersections, erosion of large motion’s basin

Evaluation of predictions (Melnikov’s method)

Nonlinear resonance

Large motion

Small motion
The roll motion (cont’d)

Integrity diagrams (quintic restoring)

Effect of bias (cubic restoring)

Transient capsize diagram

Forcing amplitude $F$

Integrity diagrams (quintic restoring)

Effect of bias (cubic restoring)

Transient capsize diagram
Stochastic rolling: wave groups

Spectrum-compatible method of wave group construction  

Markov chain model with cross correlations between heights and periods

Transition probabilities \( \rightarrow \) most expected successions of height and period in group

Karhunen–Loève representation of wave elevation

\[
\eta(x, t) = \sum_{n=0}^{\infty} a_n f_n(x, t), \quad -T < t < T
\]

*Sclavounos 2012, Proc. R. Soc. Lond. A468*

Stability assessment in two parts:

Stochastic modelling  
high probability realistic  
wave group profiles

Ship motion simulations  
using derived waveforms

Transient capsize diagram for regular  
and for realistic wave groups

The roll motion (cont’d)
The following/quartering sea

The broaching-to instability

✓ Uncontrolled, rapid deviation from desired course, sometimes ending with capsize

A topic with long history: reports date to 1699
(2010) Trans. RINA 152

“The shipwreck”
Pollock (1810)
(© National Maritime Museum, Greenwich)
inspired from Falconer (1762)

Distinctive mechanisms  JSR 1995, 40/3
- via surf-riding  more common
- resonant yaw (“direct broaching-to”)

Early scientific investigations:

Davidson (1948); Weinblum & StDenis (1950); DuCane & Goodrich (1962); Grim (1963), Wahab & Swan (1964); Boese (1970); Eda 1972; Motora et al (1982)
How surf-riding behavior comes about?

Evidence from experiments

Kan (1990), NAOE, 28, JSNA

Outcome of effort to escape from surf-riding by reducing speed or by changing heading

$F_n=0.30$

$F_n=0.35$

$F_n=0.36$
Simple analogues of surf-riding

Always downward rolling
Possible rolling
Ball trapped

Only rotations
Possible rotations
Fixed location

\[ U(x) = -\mu \cos x + \alpha \]

Grim 1983, Schiffstechnik
The following/quartering sea (cont’d)

Transition from Surging to Surf-Riding in following waves at a saddle connection

Co-existing stable states in red: cycle and fixed point

Stable state at wave trough

Surf-riding

Cycle lost at connection

Saddle connection

Unstable state at wave crest

Surge at low P in overtaking waves

From Surge to Surf-riding at high P

Propellor speed, P  Ship speed, S  Ship position, D
Surf-riding in nonlinear waves (Fenton’s variant of Stokes wave theory)

Comparison of thresholds: linear versus nonlinear wave

Continuation result: homoclinic saddle connection

Stability Diagrams: \( d = 100 \) m

Higher threshold (homoclinic)

First threshold (saddle-node)

Kontolefas (2013), NTUA Diploma Thesis

Japanese fishing vessel

The following/quartering sea (cont’d)
Coupled motions

Yaw/ sway

Period doublings leading to chaotic surf-riding

(1996) CHAOS, 6/2
Surging and surf-riding in multi-chromatic seas

Spatiotemporal framework

1-frequency excitation

2-frequency excitation

Multi-frequency excitation

“Catch and release” behavior

(2016) 31st SNH
Nonlinear surge motions - bi-chromatic excitation

Attracting (blue) and repelling (red) LCS via forward and backward FTLE fields

\[
\begin{array}{c|c}
L & 154 \text{ m} \\
B & 18.8 \text{ m} \\
T & 5.5 \text{ m} \\
m & 8858.38 \text{ t} \\
U_s & 12 \text{ m/s} \\
\end{array}
\]

(2016) 31\textsuperscript{st} SNH
Continuation result: arrangement of steady surf-riding states

Comparative result from another ship with sharper separation of dynamics

The following/quartering sea (cont’d)
Bi-chromatic excitation (cont’d)
Attracting (black) and repelling (grey) LCS - JONSWAP spectrum

Behavior in irregular seas

Converting excitation from regular to irregular
Computation of local celerity

Tri-chromatic linear wave in time-space

Lines of constant wave slope

Surge velocity and instantaneous celerity

Celerity at nearest wave max slope
The “feature flow field” method

Theisel & Seidel (2003) VisSym03

curves of minimal change of velocity in phase-space

\[ V(x_1, x_2, t) = \nabla \dot{x}_1 \times \nabla \dot{x}_2 \]

Intermittent intervals of surf-riding in time-space

LCS combined with “feature” tracking

(2016) Ocean Engineering, 120
Maneuvering and course-keeping in strong wind

Wind steering diagram for tanker in ballast

End of self-sustained oscillations in bow wind by homoclinic connection

Scenario with sluggish rudder control
Competing turning and straight-line motion

Maneuvering and course-keeping in strong wind (cont’d)

Homoclinic to saddle-node

\[ x/L \]

\[ y/L \]

\[ \theta_w = 0, \ U_w = 18 \text{ m/s} \]
Final remarks

✓ Homoclinic phenomena are quite widespread in ship motions.

✓ Their investigation in multi-chromatic seas is the state-of-art topic, enabled by recent advances in computational techniques.

✓ Efforts to incorporate more detailed mathematical models should be strengthened.

✓ Excellent prospect for developing rigorous probabilistic assessments of ship stability-related problems.

✓ Efficiency issues exist due to combination of complex physics with rarity.

✓ Need to include relevant courses in naval architecture curricula.